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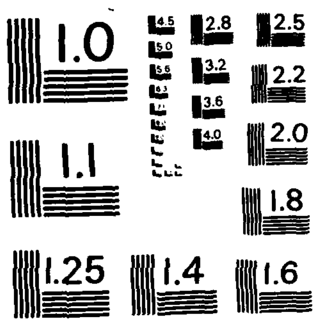

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TECHNICAL REPORT ARBRL-TR-02526

INVERSION OF RINGDOWN DATA FOR A ROTATING LIQUID CYLINDER

Keith D. Aldridge
York University
Ontario, Canada

September 1983

US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) (VAX/bja)

Disturbance pressure data from both numerical and gyroscope experiments on ringdown of inertial waves were fitted to a sum of complex exponentials using a linearized least squares method. For the numerical experiments, frequencies and decay rates agreed with those predicted by eigenvalue calculations to within a few percent. Eigenfrequencies calculated from gyroscope data

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agreed with prediction to within 2% while all decay rates were greater than predicted, some by as much as 65%.

An autoregressive method was developed and tested on both simulated and real data. At its present stage of development this approach was found to be inferior to the least squares method.

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I. INTRODUCTION

Two different approaches to the analysis of a sum of exponentially decaying sinusoids were studied in this report. The first, and what is now considered the classical method, was to minimize the mean square difference between the data and the expected response after linearization about some initial estimate point for each of the parameters. This method was used most recently by Stergiopoulos[1] in the analysis of decaying inertial waves excited in the fluid contained in a rotating cylindrical cavity both during spin-up and in the steady state. Details of this method are given in Section II and its implementation is described in Section III. The second method, called the Prony technique, linearizes the sum of exponentially modulated sinusoids by means of a recursive definition of these functions and thus concentrates the nonlinearity into a problem of finding the complex zeros of an even ordered polynomial. Background for this method is given in Section IV and its implementation is given in Section V.

The data in this exercise came from two sources. First, several time sequences were obtained from numerical simulations of the experiments of Stergiopoulos and Aldridge.[2] Results for this work are given in Sections IIIB and VB along with comparison to previous work by Sedney, Gerber, and Bartos.[3] A second source of data was from the gyroscope experiments of D'Amico.[4] Results of this work are given in Section VIB. All programs were left under the username ALDRIDGE in executable form so that any use of these programs should not require retyping into a file. A glossary of these programs is given in the Appendix.

II. LINEARIZED LEAST SQUARES

Here we give the details for the recovery of frequency, decay rate, amplitude and phase from a data set which is modeled as a sum of exponentially modulated sinusoids.

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1. *S. Stergiopoulos, "An Experimental Study of Inertial Waves in a Fluid Contained in a Rotating Cylindrical Cavity During Spin-Up From Rest," Ph.D. Thesis, York University, February 1982.*

2. *S. Stergiopoulos and K.D. Aldridge, "An Experimental Study of Complex Eigenfrequencies of Non-Axisymmetric Inertial Waves in a Rotating Fluid Cylinder During Spin-Up From Rest," Manuscript in preparation.*

3. *R. Sedney, N. Gerber, and J.M. Bartos, "Oscillations of a Liquid in a Rotating Cylinder," AIAA 20th Aerospace Sciences Meeting, AIAA-82-0296, January 1982. See also BRL Technical Report ARBRL-TR-02489, May 1983 (AD A129094).*

4. *W.P. D'Amico, Jr., W.G. Beims, and T.H. Rogers, "Pressure Measurements of a Rotating Liquid for Impulsive Coning Motion," AIAA 20th Aerospace Sciences Meeting, AIAA-82-0246, January 1982.*

Let the sequence of N observations $x(n)$ at each of a series of times $n \Delta t$ be modeled as a sum of M modes:

$$x(n) = \sum_{j=1}^{j=M} A_j \sin(\omega_j n + \phi_j) e^{-\alpha_j n} + \varepsilon(n), \qquad n = 1,2...N,$$

where $\varepsilon(n)$ is noise of zero mean and finite second moment distribution. Note that we do not assume the noise is Gaussian.

The parameters

$$A_j, \ \omega_j, \ \phi_j \text{ and } \alpha_j, \qquad j = 1,2...M$$

for the M modes are unknown and to be estimated from the time sequence $x(n)$. The calculated response based on estimates

$$\hat{A}_j, \ \hat{\omega}_j, \ \hat{\phi}_j \text{ and } \hat{\alpha}_j, \qquad j = 1,2...M$$

is given by:

$$\hat{x}(n) = \sum_{j=1}^{j=M} \hat{A}_j \sin(\hat{\omega}_j n + \hat{\phi}_j) e^{-\alpha_j n}, \qquad n = 1,2...N.$$

Our job is to estimate the parameters in a least squares sense or otherwise. We choose to minimize

$$\varepsilon = \sum_{n=1}^{n=N} \{x(n) - \hat{x}(n)\}^2$$

where N is the number of observations. If we minimize with respect to the parameters:

$$\hat{A}_j, \ \hat{\omega}_j, \ \hat{\phi}_j \text{ and } \hat{\alpha}_j, \qquad j = 1,2...M$$

then a set of nonlinear relations would result. Hence we linearize the expected response about some point represented by the superscript o so that the calculated response for the j^{th} mode at time n becomes:

$$\hat{x}(\hat{A}_j, \hat{\omega}_j, \hat{\phi}_j, \hat{\alpha}_j; n) = \hat{x}(\hat{A}_j^o, \hat{\omega}_j^o, \hat{\phi}_j^o, \hat{\alpha}_j^o; n) + (\hat{A}_j - \hat{A}_j^o) \frac{\partial \hat{x}}{\partial \hat{A}_j} +$$

$$(\hat{\omega}_j - \hat{\omega}_j^o) \frac{\partial \hat{x}}{\partial \hat{\omega}_j} + (\hat{\phi}_j - \hat{\phi}_j^o) \frac{\partial \hat{x}}{\partial \hat{\phi}_j} + (\hat{\alpha}_j - \hat{\alpha}_j^o) \frac{\partial \hat{x}}{\partial \hat{\alpha}_j} + \ldots$$

where the partial derivatives are evaluated at the estimation points denoted by the superscript o. Summation over the M modes gives the total linearized response at time n. Higher order terms are ignored and this will only lead to difficulties if the initial estimates of the parameters are too far off the actual values. Substitute this expression for x into the previous expression for the error ε and let

$$\frac{\partial \hat{\varepsilon}}{\partial \hat{A}_j} = \frac{\partial \hat{\varepsilon}}{\partial \hat{\omega}_j} = \frac{\partial \hat{\varepsilon}}{\partial \hat{\phi}_j} = \frac{\partial \hat{\varepsilon}}{\partial \hat{\alpha}_j} = 0 \qquad j = 1,2\ldots M.$$

This yields 4M equations for the 4M unknown parameters. The M equation obtained from the A_j equation are

$$\frac{\partial \hat{\varepsilon}}{\partial \hat{A}_j} = \sum_{n=1}^{n=N} \{x(n) - \hat{x}^o(n) - (\hat{A}_j - \hat{A}_j^o) \frac{\partial \hat{x}}{\partial \hat{A}_j} - (\hat{\omega}_j - \hat{\omega}_j^o) \frac{\partial \hat{x}}{\partial \hat{\omega}_j}$$

$$- (\hat{\phi}_j - \hat{\phi}_j^o) \frac{\partial \hat{x}}{\partial \hat{\phi}_j} - (\hat{\alpha}_j - \hat{\alpha}_j^o) \frac{\partial \hat{x}}{\partial \hat{\alpha}_j} \} \frac{\partial \hat{x}}{\partial \hat{A}_j} = 0, \qquad j = 1,2\ldots M,$$

where $\hat{x}^o(n)$ represents the calculated response at the superscript o values of the parameters. After collecting terms these M equations are written:

$$(A_j - \hat{A}_j^o) \sum_{n=1}^{n=N} \frac{\partial \hat{x}}{\partial \hat{A}_j} \frac{\partial \hat{x}}{\partial \hat{A}_j} + (\hat{\omega}_j - \hat{\omega}_j^o) \sum_{n=1}^{n=N} \frac{\partial \hat{x}}{\partial \hat{A}_j} \frac{\partial \hat{x}}{\partial \hat{\omega}_j}$$

$$+ (\hat{\phi}_j - \hat{\phi}_j^o) \sum_{n=1}^{n=N} \frac{\partial \hat{x}}{\partial \hat{A}_j} \frac{\partial \hat{x}}{\partial \hat{\phi}_j} + (\hat{\alpha}_j - \hat{\alpha}_j^o) \sum_{n=1}^{n=N} \frac{\partial \hat{x}}{\partial \hat{A}_j} \frac{\partial \hat{x}}{\partial \hat{\alpha}_j}$$

$$= \sum_{n=1}^{n=N} \frac{\partial \hat{x}}{\partial \hat{A}_j} \{x(n) - \hat{x}^o(n)\}, \qquad j = 1,2\ldots M.$$

These equations can be written in matrix form:

$$B^T B\ Z = B^T R$$

where the elements of the matrices are

$$B_{nj} = \frac{\partial \hat{x}(n)}{\partial \hat{p}_j},\quad Z_j = \Delta p_j,\quad R_n = x(n) - \hat{x}^o(n)$$

$$n = 1,2,\ldots N; \qquad j = 1,2,\ldots M$$

where p represents the sequence of parameters A, ω, φ, and α and Δ stands for the update of the parameter. Thus there are 4M equations in 4M unknowns.

Sequential estimates of the parameters are obtained from the update solutions Z_j so that typically

$$Z_1 = \hat{A}_1 - \hat{A}^o_1.$$

The equations are thus solved recursively until some convergence criterion is satisfied. The solution can be represented formally as:

$$Z = (B^T B)^{-1}\, B^T R$$

It has been shown by Hamilton[5] that if we let

$$(B^T B)^{-1} = Q,$$

then the variance of the parameter estimates is given by:

$$\delta^2_{Z_i} = \frac{R^T R}{N-M}\, q_{ii}$$

where q_{ii} are the diagonal elements of Q.

Thus we obtain error estimates for each parameter as well as the parameters themselves. The off diagonal elements give the covariances of these parameters. If the covariances are near unity the parameters are highly correlated; the matrix $B^T B$ is singular and there is no solution. This is a

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5. *W.C. Hamilton, Statistics in Physical Science, Ronald Press, New York, 1964.*

problem of experimental design in the sense that the covariance matrix is determined when the model is formulated, which occurs when the experiment is designed. We shall see that in our problem the exponentials modulating low frequency sinusoids are themselves not very orthogonal to each other so that high correlations exist.

III. IMPLEMENTATION OF LEAST SQUARES METHOD

The algorithm given in the previous section was coded in a FORTRAN program entitled PARTIAL.FOR and is listed in Appendix. This program includes comment statements so that it should be self-explanatory. In order to supply confirmation that the parameters returned by the program are indeed proper estimates of the actual amplitudes, frequencies and decay rates some simulated records with known parameters were inverted. The results of these simulations are described in the next section.

A. Simulations.

The immediate application of this method was the inversion of pressure data from numerical experiments by Sedney, Gerber and Bartos.[3] For these experiments the frequency and decay rates were known approximately for several data sets available. It was also known that inversion of those records with low Reynolds numbers was significantly more difficult than those at higher Reynolds numbers. For this reason a data set was simulated with two modes of frequencies 1.27 and 0.83 radians/second and decay rates 0.45 and 0.75 sec^{-1}, respectively. In order to consider the worst case as far as the inversion was concerned both modes were given the same amplitude. Results of inversion for this data are shown in Table 1. The parameters of the input waveform are given above the horizontal line in the table; four cases of output of the program, called SIM.FOR, are listed in the table. In each case the recovered parameters are shown with error deviations returned by the inversion program in parentheses on the line below. The column labeled "Sampling" shows the number of points used in the process. The column titled "Iterations" lists the number of iterations needed to satisfy the convergence criterion. In this and in most other cases this criterion was that two subsequent outputs differed by less than 1 part in 1000. Noise was added to the last case and this will be discussed below.

In principle only 9 data points should be needed to invert this data set since there are 8 parameters. (Phase is not listed in the table.) In progressing from case 1 to case 3 the number of points used in the inversion was decreased from 1000 to 50. Even with as few as 50 points estimates returned by the program were within the error estimates themselves, which were still small. With fewer points divergence occurs and this is due to round off error in the inversion process. It might seem surprising that so many more points than the minimum were needed for convergence. Not shown in this example, however, is the variance-covariance matrix of the inversion. Later it will be demonstrated that this need for so many points is essentially due to the high correlation which exists between various parameters in this problem. An error in one produces an error in another parameter. Put another way, decaying exponentials are not very orthogonal to one another. As frequency becomes

small compared to decay rate and as the number of modes in the inversion increases, this situation becomes worse.

The effect of noise on the inversion is shown in case 4. Uniformly distributed pseudo-random noise was generated and added to the simulated records. The added noise had zero mean and deviation of 10% of the simulated record. Even this modest amount of noise produces catastrophic results in the inversion. That convergence did indeed occur, but to values significantly different from the input values, is at first somewhat disturbing. This situation is less bothersome, however, when one realizes that the the pseudo noise generated by the machine is indeed correlated to some extent in violation of our initial assumption of uncorrelated noise.

A more practical problem for the inversion of numerical simulations or real experimental data is that it is usually not known how many modes are present in the record. This effect was considered in the simulations as follows. A record was simulated with no added noise using the same frequencies, decay rates and amplitudes as in Table 1. It was assumed for the inversion that only one mode was present in the simulated record. Clearly the second mode which was of equal amplitude would act as "noise" for the inversion.

The results of this test are shown in Figures 1 and 2 which are graphs of recovered frequency and decay rate during ringdown. Each point is plotted in time at the centre of a 100-point (2-second) interval over which the inversion took place. It is clear from the figure that after about 800 points both decay rate and frequency approach the initial values for the mode recovered. The interpretation of this result is that until the faster decaying mode damps out erroneous values of both frequency and decay rate occur. A practical approach to the inversion of real data where the number of modes is unknown appears to be to track the inversion process along the record in this manner. Values of frequency and decay rate at the end of the record for the one mode recovered can then be used in conjunction with estimates of parameters for a second mode. It was noted above that more than the theoretical minimum number of data points is needed to invert these data sets. This can result in divergence if too few data points are used in the data set. It was found that no matter how slowly the number of data points was decreased from a number which allowed convergence eventually divergence would occur. The example of Table 1 was modified in several ways in an attempt to alleviate this problem. The only effective change which allowed fewer data points without divergence was increasing the frequency of the modes in the simulated data. This clearly had the effect of separating the modes out to allow inversion with fewer data points. Confirmation of this hypothesis came from looking at the variance-covariance matrix of the parameters in the vicinity of divergence due to small numbers of data points. Some correlation coefficients came within unity to 1 part in 10,000 just before overflow, which is, of course, guaranteed if these cofficients are equal to 1 because the coefficient matrix, B^TB, is singular in this case. What this means to the linearized least squares method is that the signal-to-noise ratio must be increased to counter the lack of orthogonality in cases where the decay rate is comparable to the frequency. This problem is inherent in the type of function being used in the model which is, of course, determined by the experiment being carried out. The final solution to this problem is to do a different experiment or use some other inversion

scheme. It is shown later the the so-called Prony method has the potential to resolve this problem.

B. Results for Numerical Experiments.

Data from numerical experiments by Sedney, Gerber and Bartos was inverted to find best estimates of frequency and decay rate in the state of solid body rotation for some axisymmetric inertial waves. These numerical experiments were carried out for several Reynolds numbers ranging from 50 up to 1000. As mentioned above, some difficulty was experienced at low Reynolds numbers in resolving modes closely neighboring in frequency. The data used in their experiments and also in this study were axial disturbance pressure differences between the base and midplane of the cylinder. This pressure difference is labeled Column 3 in the discussion below. Also available from this previous work was the pressure difference between the base and a point at 1/4 of the cylinder's height, labeled Column 1 below. Finally, a third field was constructed from 1/2 of Column 3 less Column 1 in order to remove the response of modes with axial wave number 2.

Shown in Table 2 are the estimates of parameters for the modes excited in the numerical experiments using the linearized least squares method. Column 1 in this table lists the name of the file which was the source of pressure data. The names of these files contain two or three digits which give the perturbation frequency multiplied by 100. The first file is encoded with the Reynolds number = 50. The Reynolds number used in the numerical experiment is given in Column 2. The number of points used in the inversion is given in Column 3. Column 4 lists the number of the pressure field column as described above. Finally, Columns 5 through 8 give the estimates of amplitude, frequency, phase and decay rate for the inversion. Below each of these values is an error estimate, shown in parentheses, returned by the inversion procedure. In those entries, with two rows for each set of points, both modes were recovered simultaneously.

The results in this table divide into two groups as a result of the spatial filtering described above. All entries in the table that are labeled 1 and 3 in the fourth column are modes with axial wave number k = 2, while those labeled 2 are modes of axial wave number k = 4.

In general, there is agreement between the frequencies and decay rates shown in this table with those calculated by SGB. Exceptions to this, of course, are those results which were not in common to SGB and this report. This situation arose when other modes at nearby frequencies were found to be excited when forcing took place at a particular frequency under study.

It is important to note here, in comparison of the results in this table with those of SGB, what is meant by the error estimates given in the table. Implicit in these estimates is the integrity of the model. In particular, the noise must be uncorrelated and all modes must be found. If this is not true bias will be introduced to both frequency and decay rate estimates. This phenomenon is illustrated in the simulations of Section III-A and in the analysis of gyroscope data in Section VI. In the experience of these calculations this effect is more important in the decay rate estimates than in those for frequency.

Within the provisions given above, frequencies and decay rates given in the table are in agreement with those found by SGB. Exceptions to this are the frequencies and decay rates for the case DATA125, Reynolds number = 1000, Column 2 pressure field. The reason for this is unknown at present. In order to see this problem in more detail, the recovery of the parameters for this case and the case for Column 3 are described below.

Shown in Figure 3 is the disturbance pressure signal for the numerical experiment from DATA125, Column 3 pressure field. The abscissa is the time in seconds since the perturbation stopped, which is at time zero. There is some indication of unusual behavior in this signal near the point where t = 8 seconds. Plotted in Figure 4 is the difference between the pressure signal of Figure 3 and that calculated at each point in time using the eight parameters from the inversion in Table 2. The large spikes near t = 8 seconds are clearly due to the inability of the decaying sinusoids to fit the anomalous pressure data at that point. Although the remaining residual is only about 2% of the maximum pressure signal, it is important to note that this noise is rather highly correlated in its appearance. Furthermore, it is somewhat nonstationary in that the statistical properties appear to be changing with time. Both of these effects are in violation of our assumptions in the model. To properly assess these effects on the recovered parameters would require further investigation.

Data from Column 2 is shown in Figure 5. Figure 6 shows the residual after only one mode has been recovered from the data. This plot shows that there is clearly some nonrandom component left so that an additional mode was sought. This mode was found and is given in Table 1 as having frequency 1.6727 rad/sec. The residual shown in Figure 7 is further reduced in size but does display the same disquieting nonrandom, nonstationary noise that was seen in Figure 4.

As mentioned above, one way to proceed from there would be to further investigate the effects of violations of our assumptions. Alternatively, one could investigate a noniterative method which would not require initial estimates of parameters. Then the uncertainty of the linearized least squares method in knowing whether all modes has been found would be removed. Such a method can be derived from a relationship among decaying sinusoids due to Prony in 1795. We call this the Prony Technique and develop it for our problem below.

IV. PRONY TECHNIQUE

In the problem of data analysis for inertial waves during ringdown in a rotating fluid we seek the best fit to data for modes of the form:

$$x(n) = \sum_{j=1}^{j=M} A_j \sin(\omega_j n + \phi_j)\, e^{-\alpha_j n}, \qquad n = 1,2\ldots N, \tag{1}$$

As noted earlier in this report any direct least squares procedure for this model will lead to a nonlinear problem since the model is nonlinear in the parameters ω_j, ϕ_j, and α_j.

Our solution to this problem was to linearize about some arbitrary expansion point and solve the resulting linear least squares problem iteratively. This does indeed work and gives parameter estimates with an error measure. It has the inherent disadvantages of an iterative method in that reasonable estimates of parameters must be given in advance or divergence might well occur.

An alternative procedure to linearization by expansion as described above is to linearize by difference equation. This means defining our model recursively and in order to show this we represent a sum of decaying sinusoids as follows:

$$x(n) = \sum_{j=1}^{j=M} A_j e^{in\sigma_j} + A_j^* e^{-in\sigma_j^*} \qquad n = 1,2...N$$

where $\sigma_j = \omega_j + i\alpha_j$ and A_j are the complex eigenfrequency and amplitude of the j^{th} mode, respectively. N is the number of points and * means complex conjugate.

Now, this expression for x(n) is linear in A_j but, of course, nonlinear in σ_j as mentioned above. As shown by Chao and Gilbert,[6] the function x(n) can be expressed as a linear combination of previous values as:

$$x(n) = \sum_{j=1}^{j=2M} S_j \, x(n-j) \qquad n = 2M+1,...N$$

where M is the number of modes used. The S_j are real quantities which are related to the σ_j through the following relationship which is due to Prony and given in Reference 6:

$$Z^{2M} - S_1 Z^{2M-1} - S_2 Z^{2M-2} \ldots - S_{2M} = \prod_{j=1}^{M} (Z - e^{i\sigma_j}) \times (Z - e^{i\sigma_j^*}).$$

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6. *B.F. Chao and F. Gilbert, "Autoregressive Estimation of Complex Eigenfrequencies in Low Frequency Seismic Spectra,"* *__Geophys. J.R. Astro-Soc, 63__, 641-657, 1980.*

Thus, once the S_j are found the σ_j can be found from the roots of the polynomial.

Suppose we have a set of data x(n) and we wish to model it as:

$$\hat{x}(n) = \sum_{j=1}^{j=2M} S_j \, x(n-j) \qquad n = 2M+1,\ldots N$$

where $\hat{x}(n)$ is a predicted value of x(n) based on the previous 2M values.

Then a least squares procedure would be to minimize

$$\varepsilon = \sum_{n=2M+1}^{n=N} \{x(n) - \hat{x}(n)\}^2 .$$

Substitute the previous expression for $\hat{x}(n)$ into this to give

$$\varepsilon = \sum_{n=2M+1}^{n=N} \{x(n) - \sum_{j=1}^{j=2M} S_j \, x(n-j)\}^2 .$$

Now to formulate the least squares problem we minimize ε with respect to each of the parameters, S_j. Thus setting

$$\frac{\partial\varepsilon}{\partial S_k} = 2 \sum_{n=2M+1}^{n=N} \{x(n) - \sum_{j=1}^{j=2M} S_j \, x(n-j)\} \, x(n-k) = 0, \qquad k = 1,2\ldots 2M,$$

gives 2M equations in 2M unknowns. Interchanging the order of summation in the above relationship gives:

$$\sum_{j=1}^{j=2M} S_j \sum_{n=2M+1}^{n=N} x(n-j)x(n-k) = \sum_{n=2M+1}^{n=N} x(n)x(n-k), \qquad k = 1,2\ldots 2M.$$

Note that the terms summed over n on both sides make up an autocorrelation of x(n); after a time shift the series is multiplied by itself term by term and then added up. Since this is a regression problem the above model is given the name Autoregressive or AR model.

V. IMPLEMENTATION OF PRONY TECHNIQUE

The system of 2M linear equations in 2M unknowns is readily solved to find the S_j for $j = 1,2..2M$. Frequency and decay rate for each pair of complex roots, $z = \exp(\pm i\sigma)$, can then be found from the earlier polynomial relationship due to Prony.

A. Simulations.

In order to test the Prony method, a simulated data set of 500 points was constructed. Two sine waves of frequencies 1.0 and 1.3 radians/second, decay rate 0.5 1/second and amplitudes 0.1 and 1.0, respectively, were added together. The algorithm to recover frequency and decay rate given in the previous section was coded as the program SIMPRO.FOR. Table 3 shows the results of this simulation. The so-called recovered values are labeled with an integer from 1 to 11. Since there are two modes in the simulated record, one might expect that only two modes would need to be sought. Clearly, what seems to be happening, though, is that better approximations to the actual values of frequency and decay rate are found as more roots are sought in the polynomial. Finally, with 11 roots, the original values are reasonably recovered. The calculation stopped at 11 modes sought because of failure in the polynomial root finder.

It is important to note two points here. First, no initial estimates were required in carrying out this calculation. Second, only roots with positive complex eigenfrequencies were retained in this table; the program SIMPRO discards both negative frequencies, decay rates and any others which are outside the inertial range. This last selective mechanism is somewhat artificial for the simulated data, but it was used in preparation for running the counterpart of this simulation program on real data.

B. Results for Prony's Method.

The algorithm in SIMPRO was adjusted to the format of the data files for the numerical experiments from BARTOS and called PRONY.FOR. A test of the method on real data was carried out on the BARTOS file DATA125, Reynolds Number 1000, 438 points and Column 3 of pressure. Results of this operation are given in Table 4. Details of the input file DATA125 are given in the top of this table. Not surprisingly, a similar approach to some limiting values of complex eigenfrequency is seen here as in the previous simulation. The last case is somewhat deviant but this occurred just before the calculation failed on the next try, i.e., 9. The values of frequency and decay rate just before this failure agree within a few percent with those for the larger mode in the last two rows of Table 2. It is important to note here that the second mode recovered by the least squares method and shown in the last line of Table 2 was not recovered using the Prony method. This mode is about 15% of the larger mode and is proportionately larger than the second mode used in the simulations of Table 3. In this sense it might at first seen surprising that the smaller amplitude mode was not recovered using the Prony method. It must be remembered, however, that there was no added noise in our simulations of Table 3. What is apparently happening here is that the lack of orthogonality

of these two modes demands a larger signal-to-noise ratio than is available in the data for the numerical experiment.

Further work on the Prony method seems warranted from the above results. In particular, resolution of modes should improve if the problem were simply transformed into the frequency domain, as in Reference 6, so that some filtering would be possible. The greatest attraction of this method is that no initial estimates of parameters are needed.

VI. FORCED CONING EXPERIMENTS

The methods developed in this work were used to invert data from experimental measurements made by D'Amico[4] on pressure response of a coning fluid cylinder. These experiments were conducted as follows. A fluid cylinder of mean radius, a = 3.1761 cm and mean half height, c = 9.9986 cm was set into solid body rotation at a rate near 83.3 Hertz. The axis of symmetry of the container which coincided with the rotation axis was then forced to "cone" at a fixed angle (0.05 degrees) to a reference direction near the vertical and at various selected rates near 4.0 Hertz. After several seconds the coning of the container was stopped while the rotation continued. While the forced coning continued and for several seconds after it was stopped pressure on the base plate of the cylinder was measured and recorded in analogue form.

The purpose of the data analysis carried out here was to determine complex eigenfrequencies from the freely decaying part of the records. Six analogue records were digitized and put into disc files on the Vax by David Hepner using a FORTRAN program written by Steve Kushubar for that purpose. The Analogue to Digital converter used in this operation had a 12-bit resolution for inputs in the range from -5 volts to +5 volts. The analogue signal levels were sufficiently smaller than this range so that the maximum digital range for the largest signal was 540 rather than 4096. Sampling of the record was at 10,000 hertz so that there were approximately 125 points per cycle for signals near the spin frequency. In most cases this was more than enough so that not every point was used in the data analysis.

Six pairs of sequentially written files were provided for experimental runs at six different coning periods. Each pair of files consisted of a pressure file and a coning pulse file. The coning pulse file consisted of a time sequence of voltages on the same time base as the pressure file. Coning pulses occurred at the rate of one per coning period so that the coning pulse file consisted of a sequence of numbers corresponding to zero voltage punctuated by an integer corresponding to a voltage pulse on the original analogue record at an interval of approximately every 2500 points.

Each record consisted of 10 seconds of data so that there were 100,000 records in each of the twelve files provided. Since sequential files are read in that manner it is necessary to read n records to get to the n + 1 record. It would have taken far too much time in the data analysis to read these files sequentially so a program called CONVER.FOR was written to read each data file and then write it to a so-called direct access file. A test program to look at the direct access file called GETTER.FOR showed that the

data had been successfully transferred. This was necessary because the direct access file could not be run by EDT. Names of the files in this work are given in Table 5.

The coning files were displayed graphically using the PLOT.FOR program in order to find the location of the last coning pulse. This gave a reference point in each pressure record for the point at which forced motion had stopped. Shown in Column 2 of the above table is the approximate point in time (1 sec = 10,000 points) of the last pulse. In two of the records there were no pulses, which showed that the entire pressure record was taken during ringdown of the inertial mode.

A. Numerical Simulations.

It was important to check the method on simulated data prior to running it on real data in order to be certain that the parameters being recovered were indeed the ones in the original data. Several data files were constructed in the format of the digitized data and converted to direct access files prior to being read by the inverting program. One such data file called HEPSIM.DAT was converted to the direct access file DIRHEP.DAT. This file was input to the inversion program called DAMICO.FOR. Results of this process are given in Table 6. Convergence to the initial values is shown in the table. The phase difference shown is due to the fact that the first few points of the time sequence were ignored when the inversion took place.

B. Results for Forced Coning Experiments.

The primary response in the recorded disturbance pressure difference on the cylinder baseplate is at a frequency near the spin frequency less the coning frequency. We call that difference the response frequency. In addition to this response there was a small signal at the spin frequency due to slight mechanical irregularities in the experimental apparatus. From the point of view of this data analysis this signal was important because it allowed us to determine both spin and response during ringdown, both of which are necessary in comparing these experimental results with predictions from theory.

Shown in Table 7 are the recovered parameters from the six sequences of disturbance pressure differences described above. Column 1 lists the names of the files which contained the pressure data obtained from the D'Amico experiments. Column 2 lists the recovered spin rates in radians/second for five of the six runs. Amplitude of the spin signal in the last run was too small to be successfuly recovered. Error deviations directly from the inversion process are given in parentheses below the spin values. Recovered frequency (radians/second) and decay rate (1/seconds) for the six runs are shown in Columns 3 and 4, respectively. The columns labeled C_r and C_i are the frequency and decay rate referred to a fixed (laboratory) frame of reference. The relationship used was C_r = 1 - response/spin and C_i = decay rate/spin.

The results shown in Table 7 were obtained in each run from a set of 1667 data points in an interval during free decay. The beginning of the interval

was shortly after the last coning pulse as given above. The end of the interval was fixed at the point which gave minimum deviation in the parameters over a series of sample lengths. Typically, this gave records about 2 to 2.5 seconds in length.

It is clear from the response and decay data that the differences in response/spin from one run to the next are at best only marginally significant while the decay/spin differences are significant enough to require further attention. For if the response were explainable by any linear theory there should be no difference in these values with the different forced coning frequencies prior to ringdown.

In order to assess the above results the first record, PRES250, was studied in greater detail. If our model is correct then it should not matter what part of the interval during free decay was used for the inversion. Accordingly, each of the parameters spin, response and decay listed in Table 7 were recovered in subintervals of length 0.5 second starting every 0.1 second from 0.15 second to 1.65 seconds after the last coning pulse at 0.35 second.

A history of the spin during ringdown in the above sense is shown in Figure 8. Each point is the recovered spin frequency (radians/second) for a 1667-point interval centered at the time shown on the abscissa. The vertical bars are the standard deviation from the inversion process. Full scale on this figure is approximately 1% so that variations in spin rate during ringdown appear to be less than about 0.5%.

Simultaneously recovered in the above process was the response frequency at each point. For the purpose of comparison both in variability and deviation, recovered frequency is plotted in Figure 9 on the same vertical scale as Figure 8. Both variation of the spin frequency and its deviation are about .02% during ringdown. These small variations in response frequency are seen in Figure 10 which is the same data as in Figure 9 but with the vertical scale expanded to about 0.1% full scale. The response frequency is essentially constant during ringdown.

Had we not included the spin recovery along with the response recovery in these inversions, large apparent variations in the observed parameters would have occurred during ringdown. This is best illustrated by examining recovered decay rates in the manner of subintervals described above under two different conditions. First, we look at the decay rates found from the inversion process when only the response is recovered. In this case the existence of the spin signal, which is about 10% of the response at the beginning of ringdown, is ignored. Large variations in the decay rate are found as illustrated in Figure 11. The deviations in this figure are relatively large since only 500 points were used in this example.

For comparison decay rates found over the same interval but with both response and spin included in the inversion are plotted in Figure 12 on the same vertical scale as Figure 11. The fluctuations evident in Figure 11 are almost eliminated and an essentially constant decay rate is evident. The mean value over the interval is the value of decay rate listed in Table 7 for the run PRES250.

The difference in results shown in Figures 11 and 12, due to the spin, illustrates an important point regarding errors in this analysis. With spin ignored in the inversion spin becomes noise as far as the inversion is concerned. This results in two effects. First, the noise is clearly correlated in violation of our assumption of uncorrelated, zero mean noise. Second, all the recovered parameters are themselves related and this can be seen in the variance-covariance matrix of the parameters. By way of illustration of this correlation, the recovered amplitudes for the case of Figure 11 are plotted against recovered decay rates in Figure 13 along with the errors from the inversion. Even though the errors are large, it is clear from the figure that amplitude and decay rate are highly correlated. This means that errors in calculated amplitude result in errors in decay rate. So that in the case of ignoring the small amplitude spin signal, this disturbance pressure at this frequency becomes noise for the response at the free decay frequency. Hence large fluctuations in the decay rate are seen in Figure 11. From a more physical point of view, it is clear that the spin and response are separated by only about 24 rad/sec so that a "beating" will be evident in the combined effect of spin plus response. Anomalous values of decay would clearly be found in such a case if a segment of the record is inverted with only the response being modeled.

For comparison with theoretical prediction of the frequency and decay rate data of Table 7, the last two columns of that table are plotted as circles in Figure 14. These data were scaled by the values of C_i and C_r obtained from Reference 7 so that in the case of perfect agreement with theory all data points would fall at the point (1,1) shown by the X in the figure. Along with each data point are error estimates from the inversion process. (Note that the vertical and horizontal scales have been stretched differently because frequency is so much better determined than decay rate.)

All the data for this (3,1,1) mode show faster decay than theory predicts from less than 1% up to as much as 65% greater. Frequency, as measured in the laboratory frame, is only slightly but significantly less than that predicted by theory. As shown in the upper corner of this figure the Reynolds number based on azimuthal velocity and radius of the cylinder is 513,000 and the half-height to radius aspect ratio is 3.1.

Also shown in Figure 14 are the data from several other experiments carried out at York University by Stergiopoulos and Aldridge. These data include two nonaxisymmetric modes excited by a precessing lid and one axisymmetric mode excited by a perturbation in angular velocity. All of the latter experimental results indicate a slower decay than predicted. Like the nonaxisymmetric mode studied by D'Amico these nonaxisymmetric modes had slightly lower frequencies than prediction would expect.

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7. *C. W. Kitchens, Jr., N. Gerber, and R. Sedney, "Oscillations of a Liquid in a Rotating Cylinder: Part I. Solid Body Rotation," U.S. Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, BRL Technical Report ARBRL-TR-02081, June 1978 (AD A057759).*

VII. DISCUSSION

Some further interpretation of Figure 14 is required. It is noted first that the mode of plotting this data produces a differential stretching of the errors which have accumulated in the experiments and the data reduction. Those modes like the (1,2,1) which have values of C_r near zero will have much larger error estimates in this plot than those which are closer to unity such as the (1,1,1) mode. Had we chosen to compare experiment and theory in the rotating frame of reference this situation would simply be reversed. Thus there is no best way to compare but this differential stretching of error "bars" must be kept in mind for the interpretation.

It is probably significant that the best agreement between theory and experiment occurs for the axisymmetric mode. One is led to infer from this that the model used in the theory approximates the experiment better for the axisymmetric case than it does for the nonaxisymmetric case. If we then ask if there are any other observables in the nonaxisymmetric experiments that appear differently in the axisymmetric case we are reminded of the so-called mean flow. It was observed by Stergiopoulos[1] that associated with the nonaxisymmetric mode (1,1,1), the mean flow around the axis of rotation became unstable both during spin-up from rest and near solid body rotation for very small perturbations. D'Amico has not related any observables to the mean flow. Such a mean flow appears to become unstable for axisymmetric modes in a sphere only at very large perturbation amplitude. Support for the hypothesis that the mean flow leads to a slightly altered spin rate of the fluid with a corresponding shift in eigenfrequency would come from estimates of the mean flow itself.

Large variations in the decay rates are observed even within one set of experiments. For example, the recovered decay rates for the (3,1,1) mode vary by as much as 65%. The only experimental variable which was changed from one run to the next was the proximity to resonance when the coning was switched off. Unfortunately, too few runs have been processed at this point to draw any conclusions regarding any possible relationship between recovered decay rate and initial perturbation frequency. More data will be processed in the near future to determine this.

ACKNOWLEDGMENTS

I have benefited from many helpful discussions with Dr. Raymond Sedney and Mr. Nathan Gerber during the course of this work over the past four months. I am grateful to Dr. William P. D'Amico, Jr., for providing the gyroscope data from his experiments and Ms. Joan M. Bartos for carrying out the computations for generating the pressure fields from the numerical experiments. Finally, I wish to thank Major Robert M. Cahoon and Mr. Steven E. Kushubar for their help in the use of the Vax 11/780 which served to carry out the computations and word processing in this report.

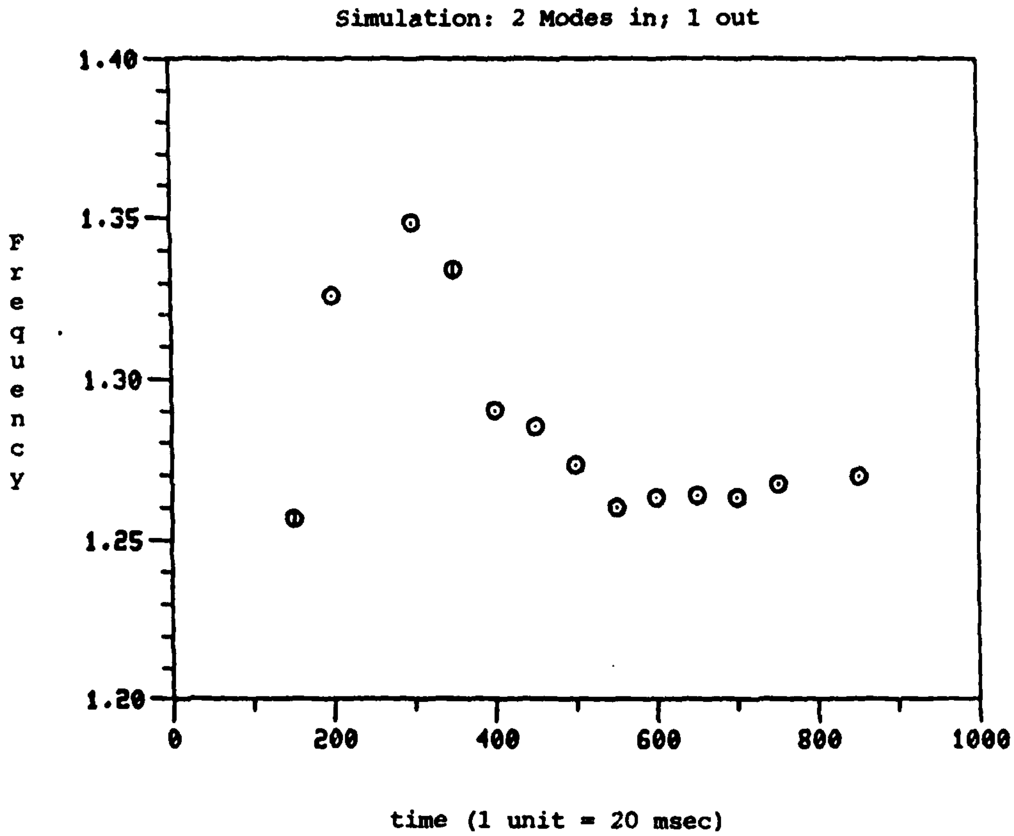

Figure 1. Effect Of Ignoring The Existence Of A Second Mode On Frequency Using Linearized Least Squares Inversion. Points Were Obtained From 100 Data Point Intervals Centered On The Plotted Location.

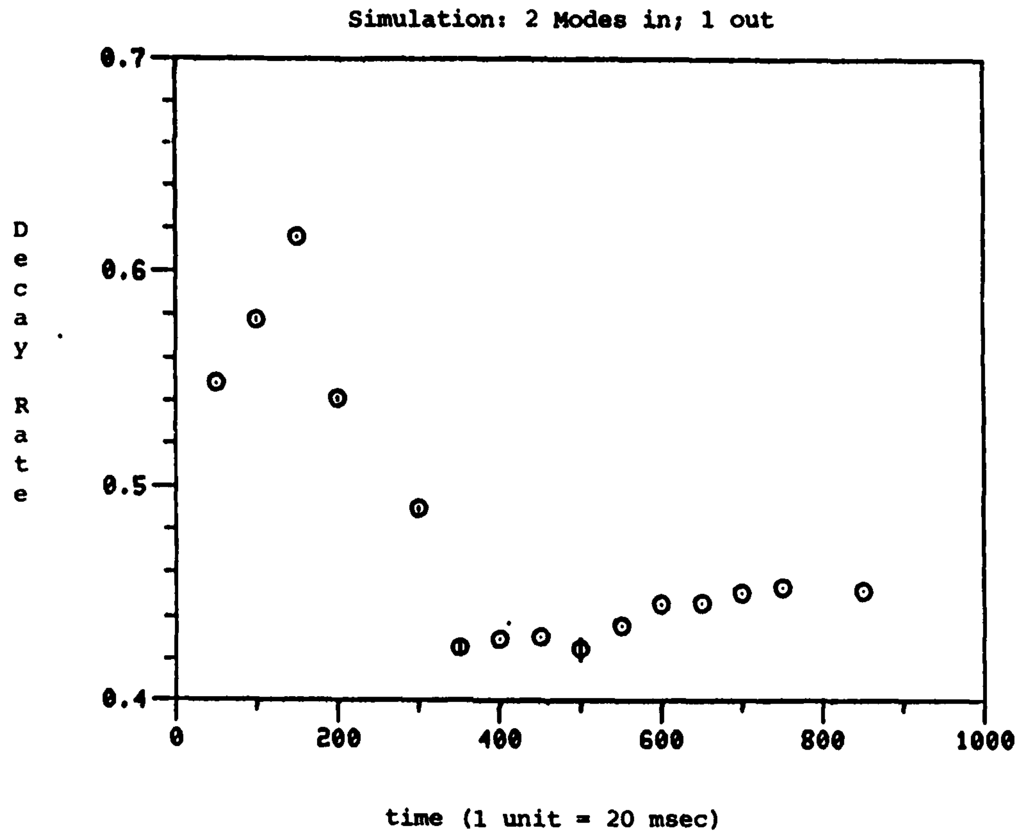

Figure 2. Same Simulation As For Figure 1 Showing The Effect On Decay Rate.

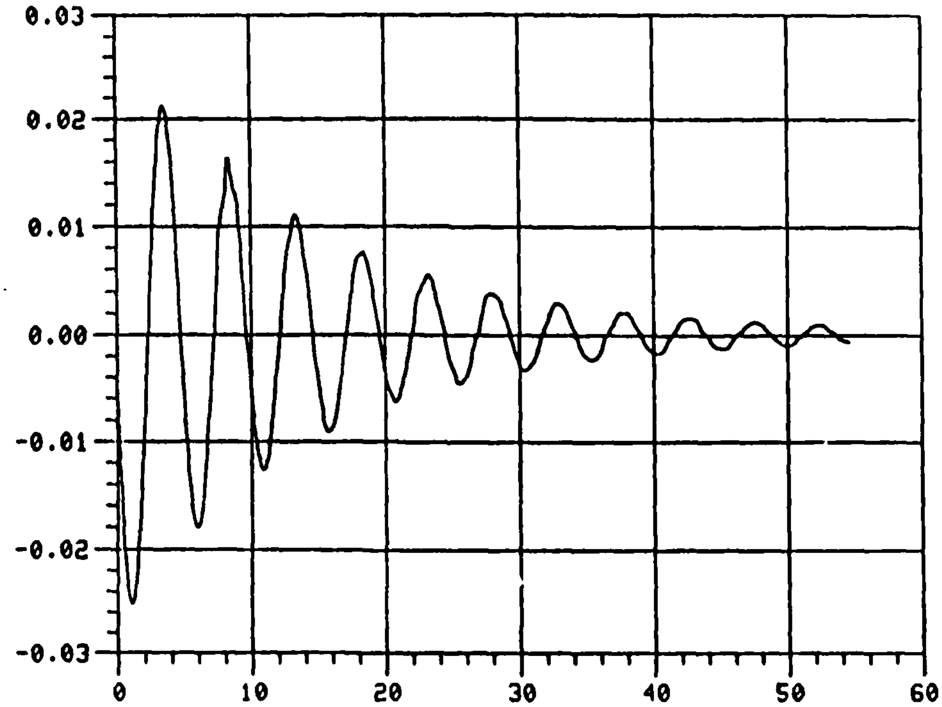

Figure 3. Axial Disturbance Pressure Difference Between Base And Center of Cylinder During Ringdown. Abscissa Is Time Since Perturbation Was Stopped In Seconds. Data Is From [BARTOS] DATA125.DAT, c/a = 1.000, Reynolds Number ≈ 1000.

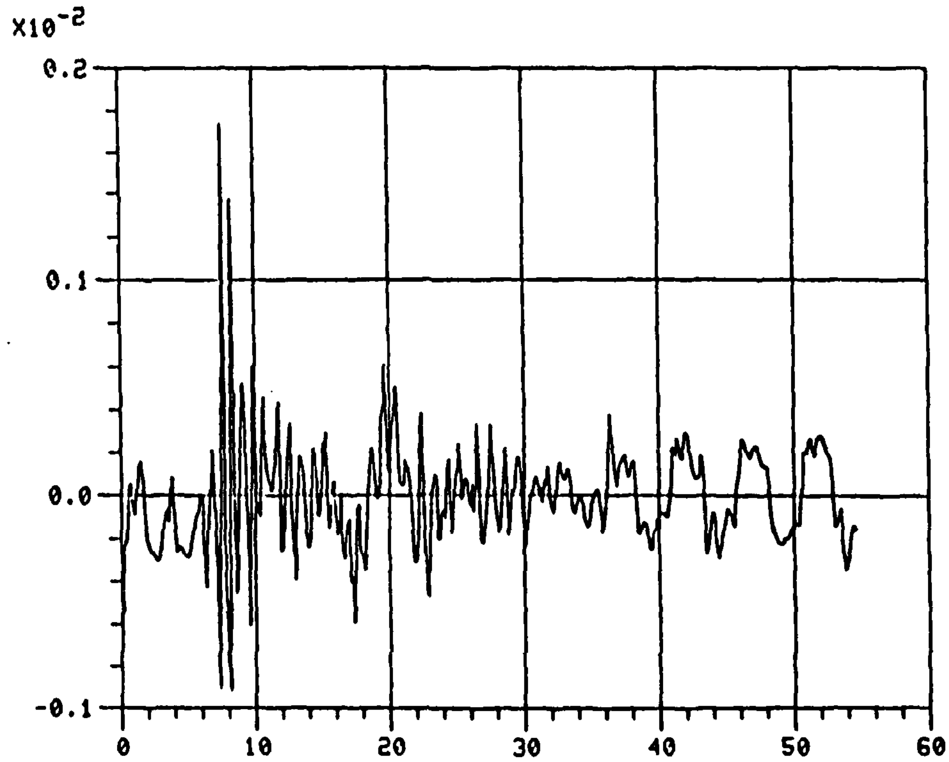

Figure 4. Residual After Convergence With 2 Modes Modeled As Listed In The Last Two Rows Of Table 2. Same Abscissa As Figure 3.

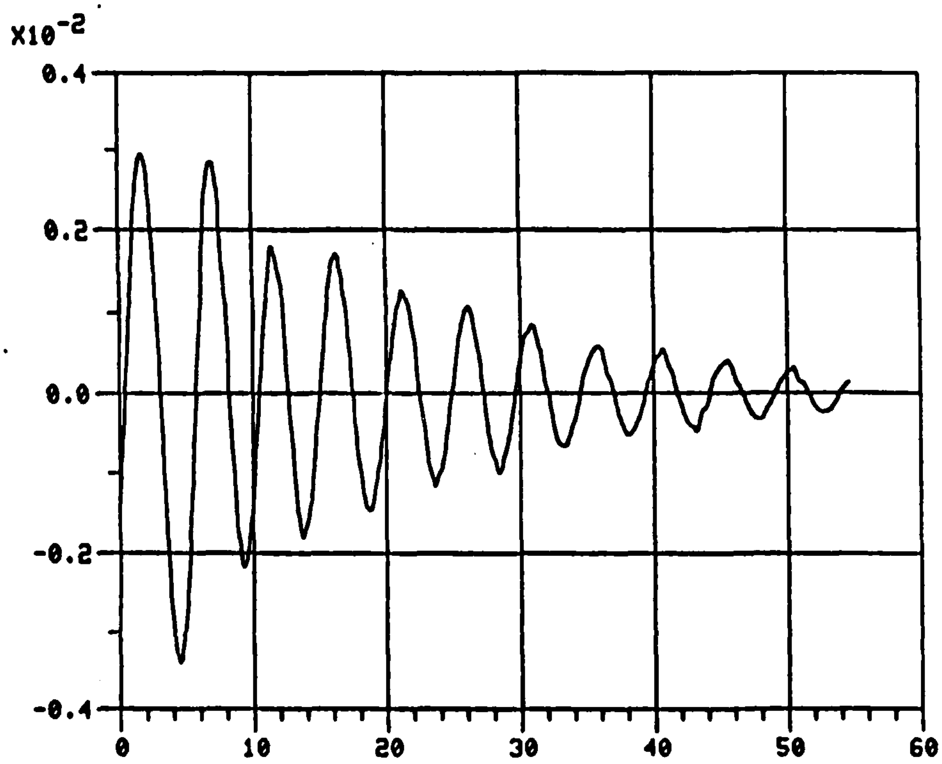

Figure 5. "Column 2" Disturbance Pressure Difference For DATA125.DAT. Same Abscissa As Figure 3.

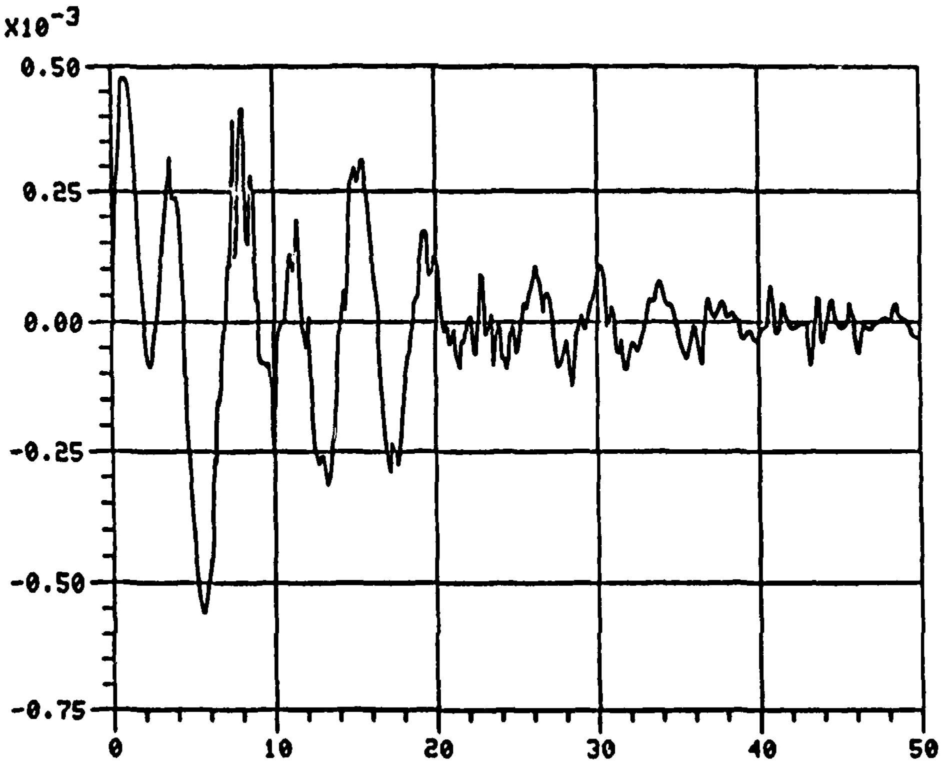

Figure 6. Residual After 1 Mode Fitted For Data Of Figure 5. Note That An Additional Mode Appears To Remain In This Residual.

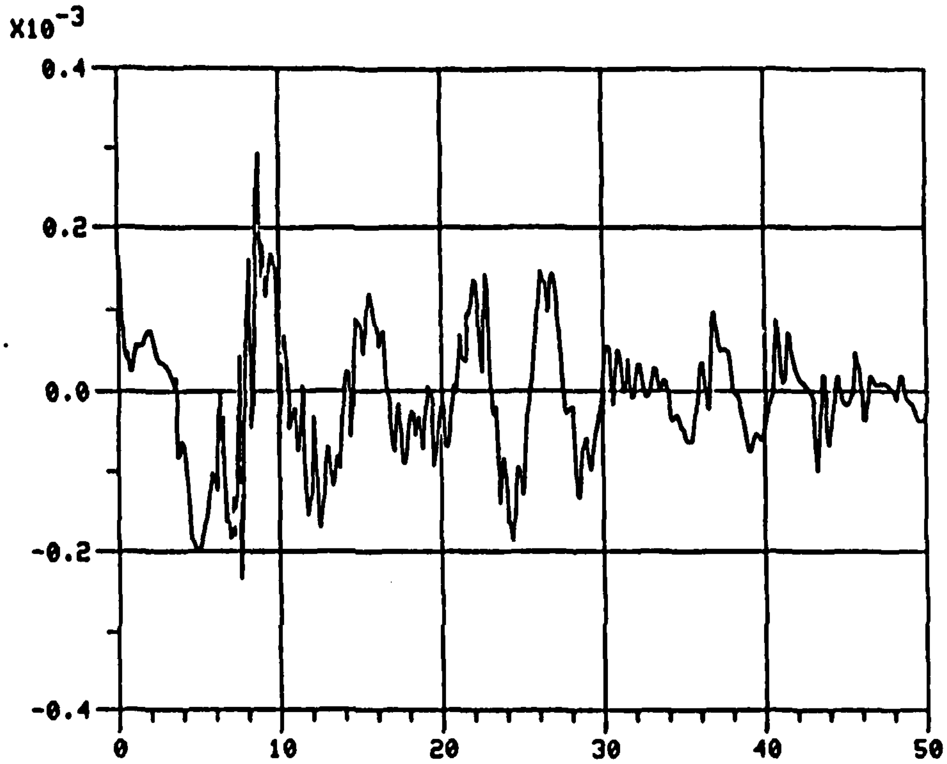

Figure 7. Residual After 2 Modes Fitted For Data Of Figure 5. Parameters For This Inversion Are Shown In Table 2, Under DATA125, 1000, 400, 2.

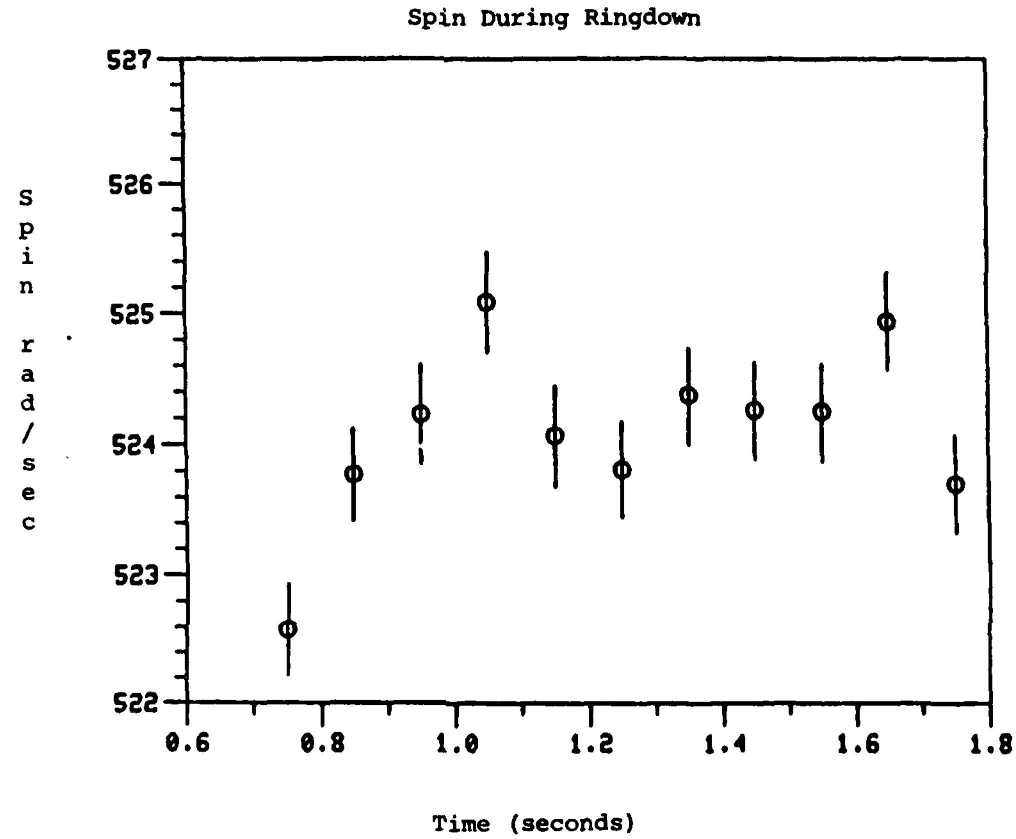

Figure 8. History Of Container Spin Rate After Coning Was Stopped For Record Originally Known as PRES250.DAT. Zero Of Time Is The Start Of Digitization.

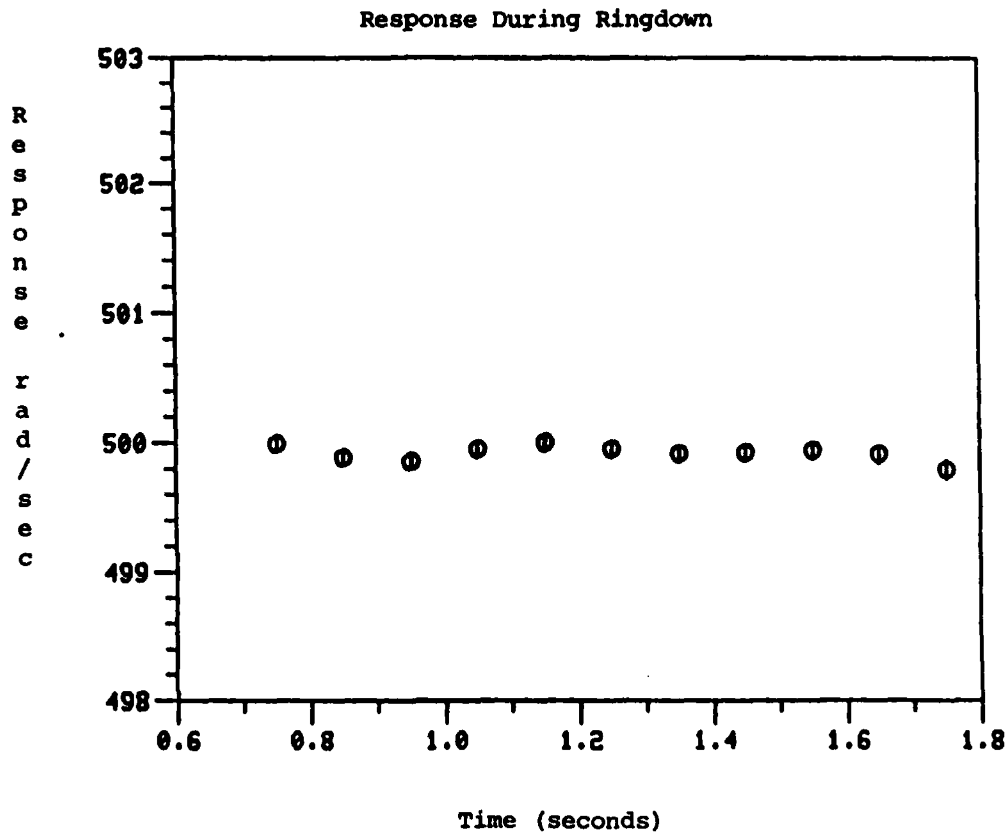

Figure 9. Recovered Response During Ringdown Plotted On The Same Vertical Axis As In Figure 8.

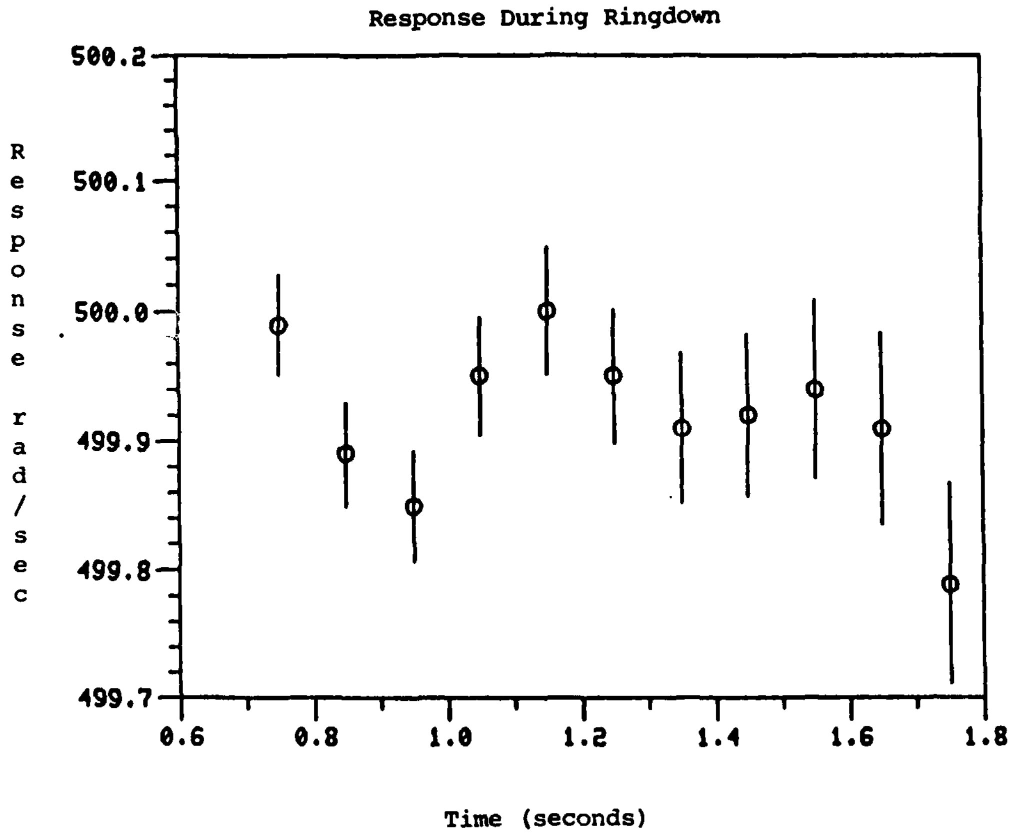

Figure 10. Data Of Figure 9 Plotted On An Expanded Scale. Note That Full Scale In This Figure Is About 0.1%.

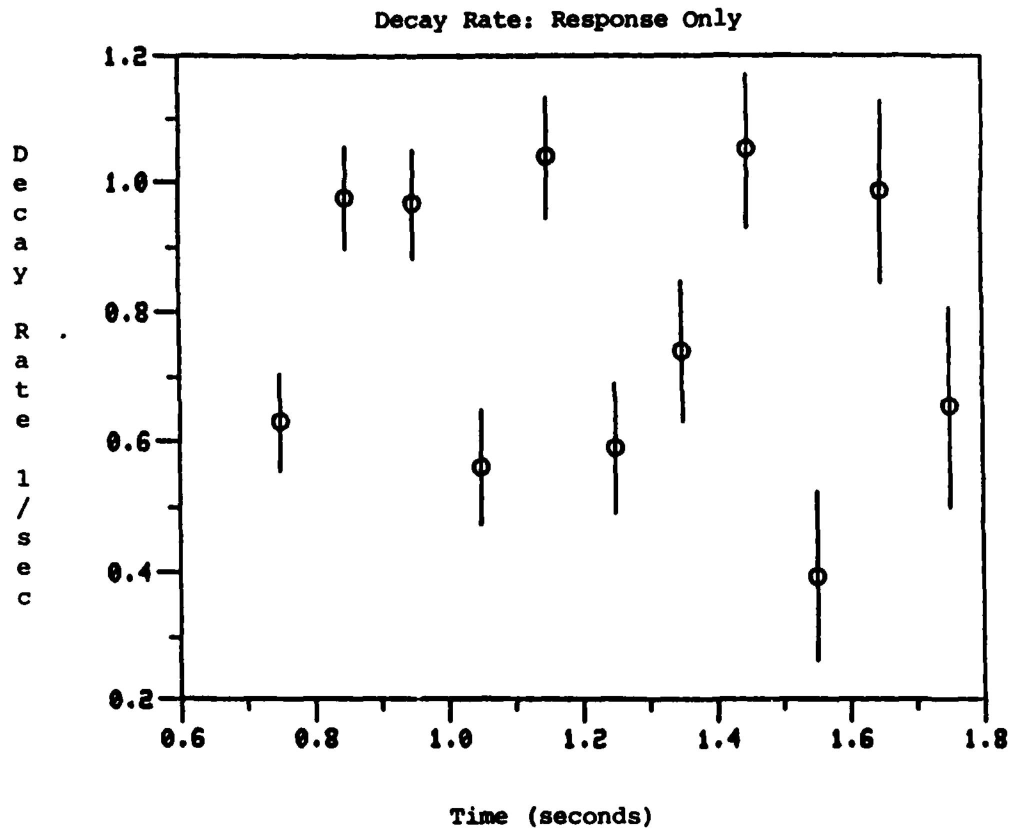

Figure 11. Recovered Decay Rates During Ringdown For PRES250.DAT Without Modeling The Spin "Noise."

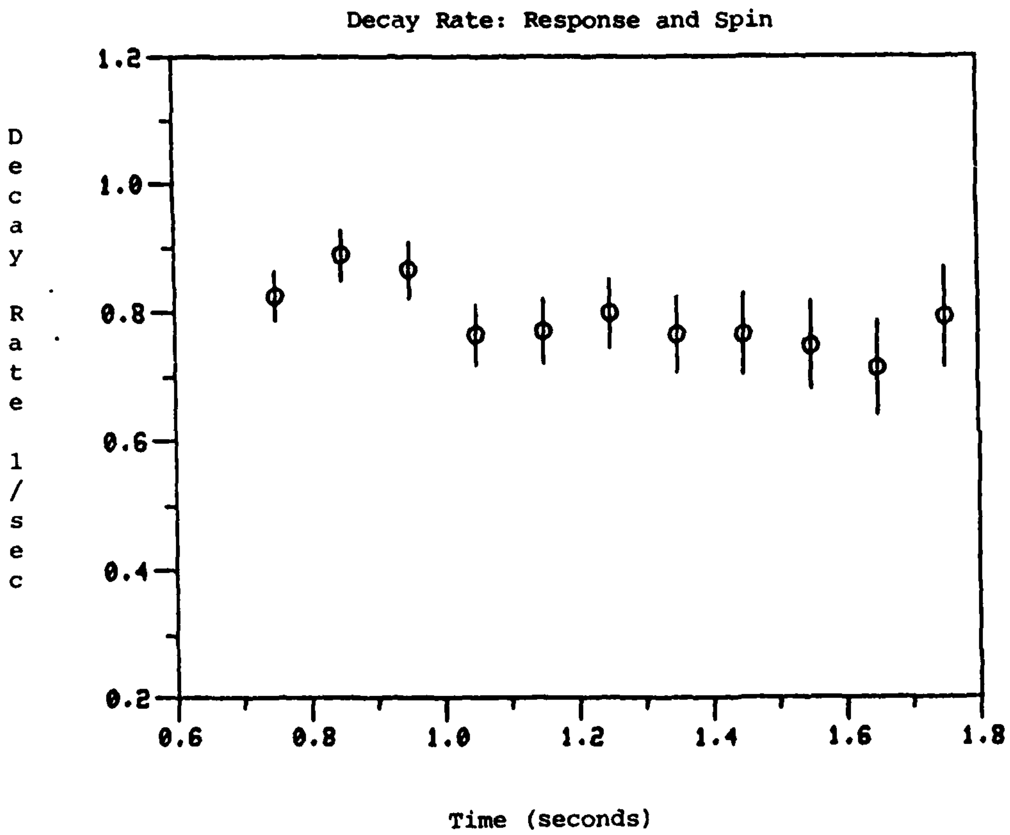

Figure 12. Decay Rates During Ringdown For PRES250.DAT With Spin "Noise" Included. Vertical Axis Is The Same Scale As In Figure 11 For Comparison.

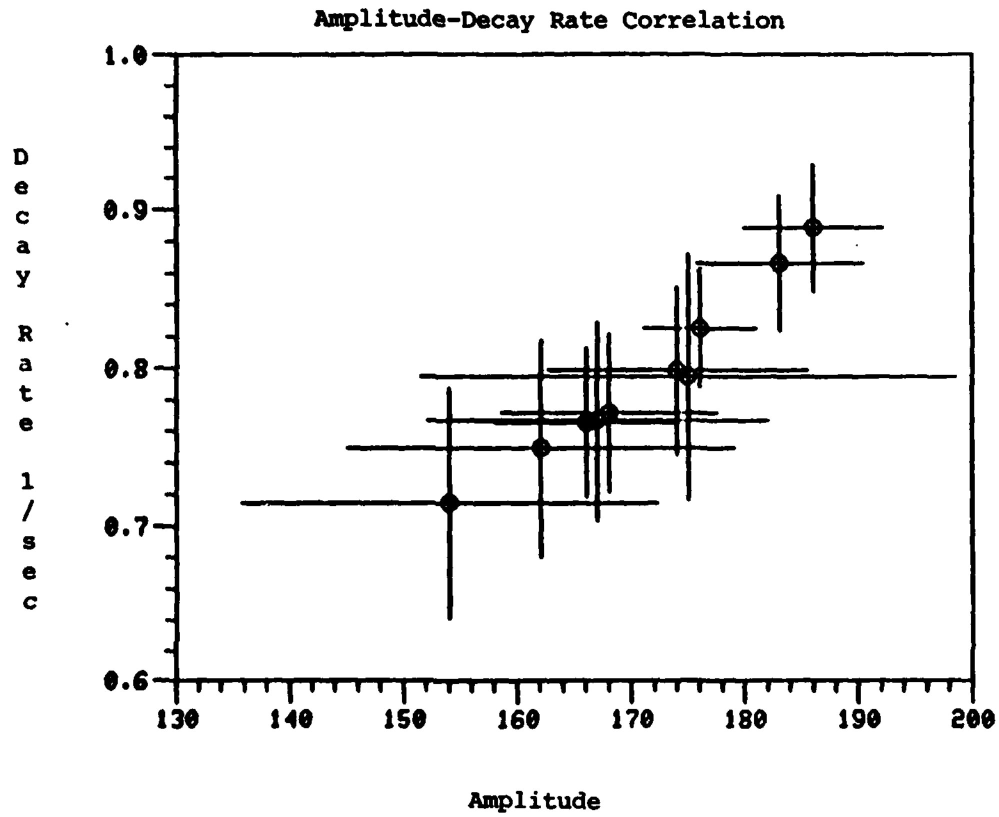

Figure 13. Demonstration Of Correlation Between Amplitude And Decay Rate For The Data PRES250.DAT.

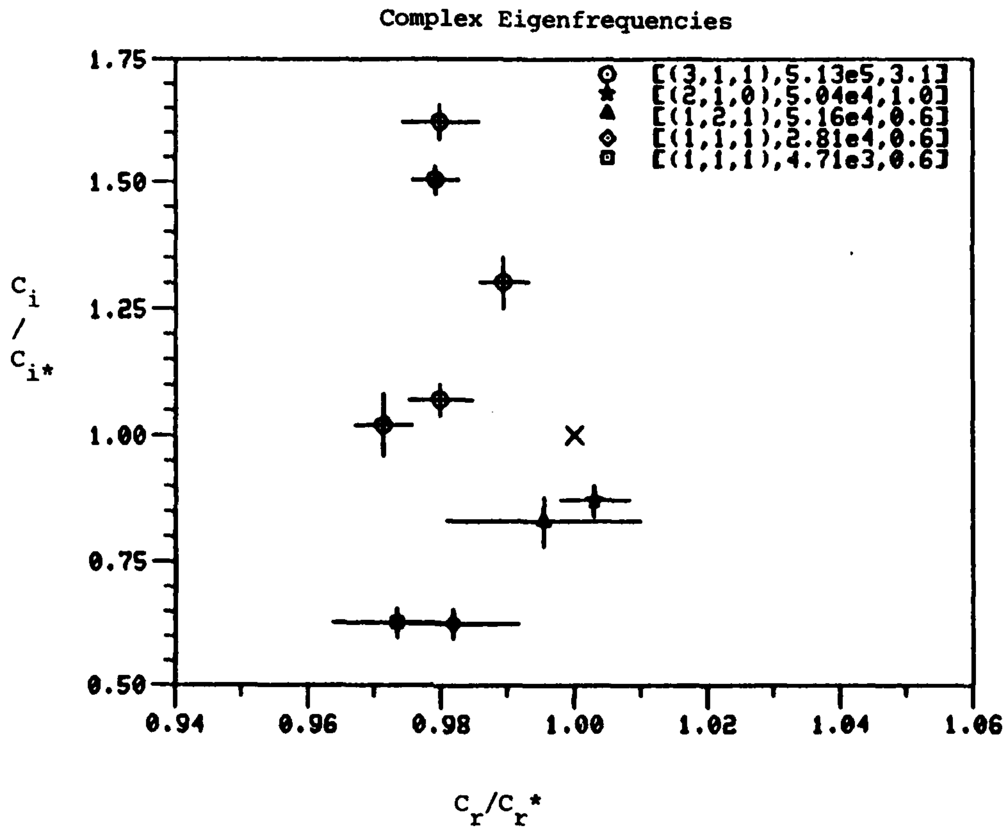

Figure 14. Normalized Real And Imaginary Parts Of Complex Eigenfrequencies For Gyroscope Experiments Of D'Amico (Circles) And Experiments Of Stergiopoulos. The * Represents Calculated Values From Reference 7. Legend Is Of Form [(k,n,m), Reynolds Number, Aspect Ratio] Where k Is Axial, n Radial, And m Azimuthal Wave Number.

TABLE 1. EFFECTS OF DATA SET SIZE AND NOISE FOR LINEARIZED LEAST SQUARES INVERSION OF TWO DECAYING SINUSOIDS.

Amplitude	Frequency	Decay Rate	Sampling	Iterations	Noise
		INPUT			
1.0000	1.27000	0.45000	100C		
1.0000	0.83000	0.75000			
		OUTPUT			
CASE 1					
1.0000	1.27000	0.45000	1000	5	
(0.00004)	(0.00001)	(0.000008)			
1.0000	0.83000	0.75000			
(0.00007)	(0.00002)	(0.00004)			
CASE 2					
1.0000	1.27000	0.45000	first		
(0.00016)	(0.00002)	(0.00004)	300	5	
1.0000	0.83000	0.75000			
(0.0001)	(0.00007)	(0.00017)			
CASE 3					
1.0242	1.2591	0.43915	first		
(0.19)	(0.069)	(0.058)	50	12	
1.0012	0.7924	0.7283			
(0.019)	(0.229)	(0.1356)			
CASE 4					
1.5155	1.2396	0.5188	1000	22	10%
(0.095)	(0.032)	(0.025)			
0.2260	0.5117	0.3777			
(0.134)	(0.185)	(0.130)			

NOTE: Numbers in parentheses are error estimates.

TABLE 2. INVERSION OF DISTURBANCE PRESSURE DATA FROM NUMERICAL EXPERIMENTS OF SGB BY LINEARIZED LEAST SQUARES.

Bartos File	Reynolds Number	Points	Col	Amplitude	Frequency	Phase	Decay
D5083	50	400	2	-0.000514	1.3947	-0.582	0.4644
				(0.000003)	(0.0026)	(0.003)	(0.0035)
DATA83B	100	1000	2	0.001587	1.2383	-0.410	0.2926
				(0.000013)	(0.0024)	(0.005)	(0.0030)
DATA117	100	1000	3	0.007066	1.1875	0.117	0.3690
				(0.000020)	(0.0011)	(0.002)	(0.0014)
DATA170	100	1000	1	0.003079	0.9462	2.847	1.2760
				(0.000129)	(0.0284)	(0.010)	(0.0229)
DATA170	100	1000	2	-0.00143	1.4044	-8.600	0.2970
				(0.00001)	(0.0034)	(0.009)	(0.0029)
DATA170	100	1000	3	-0.00310	1.2105	-0.151	0.2760
				(0.00002)	(0.0024)	(0.006)	(0.0021)
NS127Z	200	680	3	0.00857	1.3078	2.621	0.2244
				(0.00005)	(0.0006)	(0.004)	(0.0007)
				0.00547	0.9527	4.791	0.4127
				(0.00005)	(0.0021)	(0.012)	(0.0035)
NNS83Z	200	650	3	0.00382	0.8899	-0.723	0.3436
				(0.00004)	(0.0019)	(0.007)	(0.0025)
				-0.00217	1.2935	0.700	0.1801
				(0.00002)	(0.0018)	(0.013)	(0.0015)
DATA125	1000	438	1	0.00452	1.2909	2.663	0.0543
				(0.00005)	(0.0008)	(0.012)	(0.0008)
				0.00153	0.8327	4.675	0.1438
				(0.00009)	(0.0100)	(0.057)	(0.0100)
		400	2	0.00337	1.2680	3.782	0.1181
				(0.00006)	(0.0020)	(0.017)	(0.0020)
				0.00111	1.6727	0.934	0.1547
				(0.00007)	(0.0087)	(0.054)	(0.0094)
		438	3	0.01420	1.2819	2.999	0.0691
				(0.00009)	(0.0005)	(0.006)	(0.0005)
				0.00244	0.8272	4.959	0.1310
				(0.00014)	(0.0077)	(0.046)	(0.0086)

NOTE: Numbers in parentheses are error estimates.

TABLE 3. INVERSION OF SIMULATED DATA USING PRONY'S METHOD.

Delta t = 0.02000
Number of points = 500

Actual Values Used in Simulated Data

Mode	Amplitude	Frequency	Phase	Decay Rate
1	0.10000E+01	0.13000E+01	0.00000E+00	0.50000E+00
2	0.10000E+00	0.10000E+01	0.00000E+00	0.50000E+00

Recovered Values of Frequency and Decay Rate

Modes Sought	Frequency 1	Decay 1	Frequency 2	Decay 2
1	0.12763626D+01	0.50474519D+00	0.00000000D+00	0.45665833D+00
2	0.12840885D+0	0.50736495D+00	0.00000000D+00	0.12690678D+02
3	0.12845362D+01	0.50813359D+00	0.00000000D+00	0.55377725D+00
4	0.12872831D+01	0.51114615D+00	0.38778390D+00	0.14074909D+01
5	0.13004467D+01	0.50692181D+00	0.99295895D+00	0.60808377D+00
6	0.13004877D+01	0.50173362D+00	0.99754570D+00	0.52578225D+00
7	0.13003762D+01	0.50081597D+00	0.99984730D+00	0.51301232D+00
8	0.13007076D+01	0.50043101D+00	0.10044233D+01	0.51074469D+00
9	0.13004515D+01	0.50022495D+00	0.10029981D+01	0.50632917D+00
10	0.13002653D+01	0.50012187D+00	0.10017912D+01	0.50361231D+00
11	0.13000547D+01	0.50004500D+00	0.10002564D+01	0.50093839D+00

TABLE 4. INVERSION OF DATA125.DAT USING PRONY'S METHOD.

Inversion for data from DATA125:

Delta t	=	0.12500
Shutoff at:		70.37500
Frequency	=	1.25000
Number of points	=	438
Aspect ratio (c/a)	=	1.00000
Reynolds number	=	1000.00000
Data column	=	3

Points 1 to 438:

Modes Sought	Frequency	Decay Rate
1	0.12513086D+01	0.46596291D+00
2	0.12735635D+01	0.22293002D+00
3	0.12592338D+01	0.83474899D-01
4	0.12597282D+01	0.72222699D-01
5	0.12590036D+01	0.67341016D-01
6	0.12617913D+01	0.69374545D-01
7	0.12620774D+01	0.69379605D-01
8	0.12671463D+01	0.68961001D-01

TABLE 5. NOMENCLATURE FOR DATA FILES IN ANALYSIS OF GYROSCOPE DATA.

Coning File	Last Pulse	Hepner's File	Direct Access File
CONE250.DAT	3500	PRES250.DAT	DIRP250.DAT
CONE255.DAT	2000	PRES255.DAT	DIRP255.DAT
CONE266.DAT	<0	PRES266.DAT	DIRP266.DAT
CONE285.DAT	<0	PRES285.DAT	DIRP285.DAT
BCON255.DAT	5500	BPRE255.DAT	DIRBP255.DAT
BCON267.DAT	4800	BPRE267.DAT	DIRBP267.DAT

TABLE 6. DEMONSTRATION OF LINEARIZED LEAST SQUARES INVERSION USING DIRECT ACCESS FILE AS INPUT.

Mode	Amplitude	Frequency	Phase	Decay rate
Starting estimates are:				
1	0.25500E+03	0.49500E+03	0.00000E+00	0.90000E+00
Estimates after 7 iteration(s)				
	0.24942E+03	0.50000E+03	-0.24993E+00	0.10019E+01
Errors				
	0.86487E-01	0.13788E-02	0.34798E-03	0.13718E-02
Actual values used in source				
	250.00	500.00	0	1.000

TABLE 7. RESULTS FOR INVERSION OF GYROSCOPE DATA.

D'Amico File	Spin (Dev) (1/sec)	Response (Dev) (1/sec)	Decay (Dev) (1/sec)	C_r	C_i
PRESS250	524.04 (0.12)	499.96 (0.02)	0.7797 (0.02)	0.04595	0.001488
PRESS255	523.51 (0.09)	499.47 (0.02)	1.0953 (0.02)	0.04592	0.002092
PRESS266	523.66 (0.09)	499.36 (0.04)	0.9480 (0.04)	0.04640	0.001810
PRESB255	523.43 (0.10)	499.59 (0.03)	0.7430 (0.03)	0.04555	0.001419
PRESB267	523.38 (0.14)	499.33 (0.03)	1.1807 (0.03)	0.04595	0.002256
PRESS285		499.55 (0.08)	0.8841 (0.08)		

NOTE: Numbers in parentheses are error estimates.

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APPENDIX A

GLOSSARY OF PROGRAMS

APPENDIX A

GLOSSARY OF PROGRAMS

Listed below are the names of FORTRAN programs and brief descriptions of their function. All programs were left in executable form under the username ALDRIDGE, password XYZZY.

In general, the programs are self-explanatory in their input and output. One exception to this is the use of CONTROL Z on input. Generally, these programs have sequential input from one input line to the next. If one wishes to go back to the previous input line instead of entering a number on the current line, CTRL Z will usually allow this. Finally, in the iterative programs, previously calculated values of parameters will be read into the input variables by entering CTRL Z instead of the requested input. All inputs in these programs are free format so that numbers need only be separated by a space.

Numerical Experiments

PARTIAL.FOR Linearized least squares inversion program reads input from BARTOS data files. Output is sent to a file with extension name .LIS. For example: if the input file is DATA125.DAT output would appear in DATA125.LIS. Only the initial estimates and the last iteration are sent to the .LIS file.

PARSEE.FOR is the same as PARTIAL.FOR except that it displays the residual error graphically at each iteration. This is useful when first beginning the iteration scheme in order to display the data. It is also useful after a mode has been recovered to see what, if anything, is left.

SIM.FOR is essentially the program PARTIAL set up for reading in simulated data generated by GEN.FOR.

GEN.FOR generates 1 or more decaying sinusoids for input to SIM.FOR.

Prony's Method

PRONY.FOR inverts data from BARTOS files of numerical experiments using Prony's method.

SIMPRO.FOR used for testing the Prony method with data sets with known parameters generated by SIM.FOR.

Forced Coning Experiments

DAMICO.FOR linearized least squares inversion of data from direct access file of pressure data created by CONVER.FOR from sequential data files of pressure of the type PRESS250.DAT created by Hepner for D'Amico's pressure data.

CONVER.FOR reads sequential pressure file and creates a direct access file under the name DIRACC.DAT. Operator should rename this file appropriately.

GETTER.FOR reads direct access files created by CONVER.FOR and displays results to the screen. Used for reassurance that the data has been properly converted. FSTDM.FOR is the same program as DAMICO.FOR but without any graphics.

Miscellaneous

KAPOINT.FOR plots 1 or more sets of data points on a graph and a legend as well as title and axes. Sample input for this marginally documented program is CRCINM.DAT.

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